

Improved identity-based identification using correcting codes

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Abstract—In this paper, a new identity-based identification scheme based on error-correcting codes is proposed.

Two well known code-based schemes are combined : the signature scheme by Courtois, Finiasz and Sendrier and an identification scheme by Stern.

A proof of security for the scheme in the Random Oracle Model is given.

Index Terms—Identification, Identity-based Cryptography, Correcting codes, Stern, Niederreiter.

I. INTRODUCTION

ONE of the most critical points of public key cryptography (PKC) is that of the management of the authenticity of the public key. It is the very single point that anchors public key cryptography to the real world. If no such a mechanism is provided the consequences are fatal. In fact, if Alice is able to take Bob's identity by faking her own public key as Bob's one, she would be able to decipher all messages sent to Bob or to sign any message on behalf of Bob.

In 1984, Shamir introduced the concept of Identity-based Public Key Cryptography ID-PKC [27] in order to simplify the management and the identification of the public key, which, time passing by, had become more and more complex.

In ID-PKC the public key of an user is obtained from his identity id on the network. The identity id can be a concatenation of any publicly known information that singles out the user : a name, an e-mail, or a phone number, to name a few. Hence it is not longer necessary to verify a certificate for the public key nor to access a public directory to obtain a certificate. At first glance it seems simple but producing private keys becomes more complex. In particular a user can not build his own private key by himself anymore, and it is necessary to introduce a trusted third party who constructs the private key from the user's identity and sends it to the user. This process has to be done at least once for each user.

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Shamir [27] calls this trusted third party the Key Generation Center (KGC). The KGC is the owner of a system-wide secret, thus called the *master key*. After successfully verifying (by non-cryptographic means) the identity of the user, the KGC computes the corresponding user private key from the master key, the user identity id and a trapdoor function.

Identity-based systems resemble ordinary public-key systems, in the sense that both involve a private transformation (i.e. decrypting) as well as a public transformation (i.e. encrypting). However, in identity-based systems users do not have explicit public keys. Instead, the public key is effectively replaced by (or constructed from) a user's publicly available identity information.

The motivation behind identity-based systems is to create a cryptographic system resembling an ideal mail system. In this ideal system, knowledge of a person's name alone suffices for confidential mailing to that person, and for signature verification that only that person could have produced. In such an ideal cryptographic system :

1. users need not exchange neither symmetric keys nor public keys;
2. public directories (databases containing public keys or certificates) need not be kept;
3. the services of a trusted authority are needed solely during a set-up phase (during which users acquire authentic public system parameters).

A drawback in many concrete proposals of identity-based systems is that the required user-specific identity data includes additional data, taking the form of an integer or public data value for instance, denoted DA, beyond an a priori identity ID. Ideally, DA is not required, as a primary motivation for identity-based schemes is to eliminate the need to transmit public keys, to allow truly non-interactive protocols with identity information itself sufficing as an authentic public key. We will refer to the latter systems as *pure identity-based* systems. The issue is less significant in signature and identification schemes where the public key of a claimant is not required until receiving a message from that claimant (in this case DA is easily provided); but in this case, the advantage of identity-based schemes diminishes. It is more critical in key agreement and public-key encryption applications where another party's public key is needed at the outset.

In his paper Shamir proposed identity-based signature and identification systems based on the RSA or Discrete Logarithm problems. The first efficient provably secure identity-based encryption cryptosystem featuring the above mentioned non-interactive property was proposed in 2001 by Boneh and Franklin [16]. This system is based on the Weil pairing over certain families of elliptic curves. The same year, Cocks [10] published a system based on quadratic residuosity but a rather large message expansion makes it somewhat inefficient in practice.

Following the paper by Boneh and Franklin, research on ID-PKC has made great advances and lots of schemes

have been published, most of them based on elliptic curves and bilinear pairings, such as identity-based encryption (IBE) schemes [4], identity-based key agreement schemes [5], identity-based identification (IBI) or identity-based signature (IBS) schemes [9], [32], [33]. In 2004 Bellare, Neven and Namprempre proposed in [1] a general framework deriving IBI or IBS from traditional public key-based signature and identification schemes and they applied it to concrete known schemes. The resulting systems are not pure identity-based and only schemes based on number theoretic problems were considered.

In this paper, we propose and formally study a new IBI scheme built from error-correcting codes.

Code-based cryptography was introduced by McEliece [23], a variation of which was later proposed by Niederreiter [25]. The idea of using error-correcting codes for identification purposes is due to Harari [20], followed by Stern (first protocol) and Girault [17]. But Harari and Girault protocols were subsequently broken, while Stern's one was five-pass and unpractical. At Crypto'93, Stern proposed a new scheme [30], which is still today the reference in this area.

For a long time no code-based signature scheme was known, eventually the first (not yet cryptanalyzed) one was proposed by Courtois, Finiasz and Sendrier [11] (CFS) in 2001. The basic idea of the CFS signature scheme is to choose parameters such that an inversion of the otherwise non-invertible Niederreiter scheme is feasible. This is done at the cost of a rather large public key when comparing to other signature schemes. Still signature length is short.

We obtain our new IBI scheme by combining the CFS signature scheme and the identification scheme by Stern. The basic idea of our scheme is to start from a Niederreiter-like problem which can be inverted like in the CFS scheme. This permits to associate a secret to a random (public) value obtained from the identity of the user. The secret and public values are then used for the Stern zero-knowledge identification scheme.

The paper is organized as follows. In Section II we introduce notation and definitions, while in Section III we recall basic facts on code-based cryptography. Section IV is devoted to describe the public key encryption scheme of Niederreiter and the signature scheme of Courtois, Finiasz and Sendrier. The identification protocol of Stern is presented in Section V, and next our new protocol is described in Section VI. In Section VII we give a proof of security for our scheme in the Random Oracle Model [2].

Finally in Section VIII we give concrete parameters and conclude in Section IX.

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II. NOTATION AND DEFINITIONS

WE first introduce some notation. If x is a string, then $|x|$ denotes its length, while if S is a set then $|S|$ denotes its cardinality. If $\kappa \in \mathbb{N}$ then 1^κ denotes the string of κ ones.

If S is a set then $s \xleftarrow{R} S$ denotes the operation of picking an element s in S uniformly at random. Unless otherwise indicated, algorithms are modelled as Probabilistic Polynomial Time (PPT) algorithms. We write $\mathcal{A}(x, y, \dots)$ to indicate that \mathcal{A} is an algorithm with inputs x, y, \dots and by $z \leftarrow \mathcal{A}(x, y, \dots)$ we denote the operation of running \mathcal{A} with inputs (x, y, \dots) and letting z be the output. We write $\mathcal{A}^{\mathcal{O}_1, \mathcal{O}_2, \dots}(x, y, \dots)$ to indicate that \mathcal{A} is an algorithm with inputs x, y, \dots and access to oracles $\mathcal{O}_1, \mathcal{O}_2, \dots$ and by $z \leftarrow \mathcal{A}^{\mathcal{O}_1, \mathcal{O}_2, \dots}(x, y, \dots)$ we denote the operation of running \mathcal{A} with inputs (x, y, \dots) and access to oracles $\mathcal{O}_1, \mathcal{O}_2, \dots$ and letting z be the output.

PROVERS AND VERIFIERS. An interactive algorithm is a stateful PPT algorithm that on input an incoming message M_{in} (this is ε if the party is initiating the protocol) and state information St outputs an outgoing message M_{out} and updated state St . The initial state contains the initial inputs of the algorithm. We say that \mathcal{A} accepts if $M_{\text{out}} = \text{acc}$ and rejects if $M_{\text{out}} = \text{rej}$. An interaction between a prover P and a verifier V , both modelled as interactive algorithms, ends when V either accepts or rejects. The expression :

$$(C, d) \leftarrow \mathbf{Run}[P(p_1, \dots) \leftrightarrow V(v_1, \dots)]$$

denotes that P and V have initiated in an interaction with inputs p_1, \dots and v_1, \dots respectively, getting a conversation transcript C and a boolean decision d , with 1 meaning that V accepted, and 0 meaning it rejected.

STANDARD IDENTIFICATION SCHEMES. A standard identification scheme $s = (Kg, P, V)$ consists of three PPT algorithms :

Key generation algorithm Kg takes as input a security parameter κ and returns a secret key SK and a matching public key PK . We use the notation $(SK, PK) \leftarrow Kg(1^\kappa)$.

Interactive identification protocol, where the prover runs P with initial state SK , while the verifier has initial state PK . It is required that for all $\kappa \in \mathbb{N}$ and valid key pairs (PK, SK) , the output by V in any interaction between V (with input PK) and P (with input SK) is acc with probability one.

STANDARD SIGNATURES. A standard signature scheme $s = (KG, Sign, Vfy)$ consists of three PPT algorithms :

Key generation algorithm KG takes as input a security parameter κ and returns a secret key SK and a matching public key PK . We use the notation $(SK, PK) \leftarrow KG(1^\kappa)$.

Signing algorithm $Sign$ takes as input a secret key SK and a message m . The output is a signature $sig_{SK}(m)$. This is denoted as $sig_{SK}(m) \leftarrow Sign(SK, m)$.

Verification algorithm Vfy takes as input a public key PK , a message m , and a signature $sig = sig_{SK}(m)$. The output is 1 if the signature is valid, or 0 otherwise. We use the notation $\{0, 1\} \leftarrow Vfy(PK, m, sig)$ to refer to one execution of this algorithm.

The standard security notion for signature schemes is unforgeability against adaptively-chosen message attacks, which can be found in [19].

IDENTITY-BASED IDENTIFICATION. An identity-based identification scheme $\mathcal{IBS} = (\text{MKg}, \text{UKg}, \bar{P}, \bar{V})$ consists of four PPT algorithms, as follows :

Master-key generation algorithm MKg takes as input a security parameter κ and returns, on one hand, the system public parameters mpk and, on the other hand, the matching master secret key msk , which is known only to a master entity. It is denoted as $(mpk, msk) \leftarrow \text{MKg}(1^\kappa)$.

Key extraction algorithm UKg takes as inputs the master secret key msk and an identity $id \in \{0, 1\}^*$, and returns a secret key $SK[id]$. We use the notation $SK[id] \leftarrow \text{UKg}(msk, id)$.

Interactive identification protocol, where the prover with identity id runs the interactive algorithm \bar{P} with initial state $SK[id]$, and the verifier runs \bar{V} with initial state mpk, id .

SECURITY OF IBI SCHEMES. An IBI scheme is said to be secure against impersonation under passive attacks (**imp-pa**) if any adversary $\mathcal{A} = (\bar{CP}, \bar{CV})$, consisting of a cheating prover \bar{CP} and a cheating verifier \bar{CV} , has a negligible advantage in the following game :

Setup The challenger takes a security parameter κ and runs the master key generation algorithm MKg . It gives mpk to the adversary and keeps the master secret key msk to itself. It initializes an empty list UK^{list} .

Phase 1 The adversary issues queries of the form

- User key query $\langle \mathcal{ID}_i \rangle$. The challenger checks whether there exists an entry $(id_i, SK[id_i])$ in the list UK^{list} . If this is the case, it retrieves the user secret key $SK[id_i]$. Otherwise, it runs algorithm UKg to generate the private key $SK[id_i]$ corresponding to id_i . It sends $SK[id_i]$ to the adversary. It includes the entry $(id_i, SK[id_i])$ in the list UK^{list} .
- Conversation query $\langle \mathcal{ID}_i \rangle$. The challenger checks whether there exists an entry $(id_i, SK[id_i])$ in the list UK^{list} . If this is the case, it retrieves the user secret key $SK[id_i]$. Otherwise, it runs algorithm UKg to generate the private key $SK[id_i]$ corresponding to \mathcal{ID}_i . The challenger returns (C, d) where $(C, d) \leftarrow \text{Run}[\bar{CP}(SK[id_i]) \leftrightarrow \bar{V}(mpk, id_i)]$.

These queries may be asked adaptively, that is, each query may depend on the answers obtained to the previous queries.

Challenge The cheating verifier \bar{CV} outputs a target identity id^* and its state $St_{\bar{CV}}$, such that the private key for id^* was not requested in Phase 1.

Phase 2 The cheating prover \bar{CP} , with input $St_{\bar{CV}}$, interacts with a honest verifier with input mpk, id^* . The cheating prover is allowed to query the same oracles as

in Phase 1, except that the query id^* is not allowed. Finally, \mathcal{A} wins if the output of \bar{V} is accept, i.e. $d = 1$ in $(C, d) \leftarrow \text{Run}[\bar{CP}(SK[id_i]) \leftrightarrow \bar{V}(mpk, id_i)]$.

Such an adversary is called an **imp-pa** adversary \mathcal{A} , and its advantage is defined as $\text{Adv}_{\mathcal{IBS}, \mathcal{A}}^{\text{imp-pa}}(1^\ell) = \Pr[d = 1]$.

III. CODE-BASED CRYPTOGRAPHY

IN this section we recall basic facts about code-based cryptography. We refer to the work of Sendrier [26] for a general introduction to these problems.

A. Hard problems

Every public key cryptosystem relies on a hard problem. In the case of coding theory, the main hard problems used are the Bounded Decoding (BD) and Code Distinguishing (CD) problems.

Definition III.1 (Bounded Decoding Problem) Let n and k be two integers such that $n \geq k$ and H a parity check matrix. $\text{Binary}(n, k)$ represents a random binary matrix of n columns, k rows and of rank k .

Input : $H \xleftarrow{R} \text{Binary}(n, k)$ and $s \xleftarrow{R} \mathbb{F}_2^{n-k}$

Ouput : A word $e \in \mathbb{F}_2^n$ such that $\text{wt}(e) \leq \frac{n-k}{\log_2 n}$ and $He^T = s$

Let us denote by $\text{Adv}_{\mathcal{C}}^{\text{BD}}(n, k)$ the probability that an algorithm \mathcal{C} has in solving the above problem.

This problem was proven to be NP-complete in [3].

Definition III.2 (Code Distinguishing Problem)

Let n and k be two integers such that $n \geq k$ and H a parity check matrix.

Input : $H \xleftarrow{R} \text{Goppa}(n, k)$ or $H \xleftarrow{R} \text{Binary}(n, k)$.

Ouput : $b = 1$ if $H \in \text{Goppa}(n, k)$, $b = 0$ otherwise.

$$\text{Adv}_{\mathcal{D}}^{\text{CD}}(n, k) = \left| \Pr[\mathcal{D}(H) = 1 \mid H \xleftarrow{R} \text{Goppa}(n, k)] - \Pr[\mathcal{D}(H) = 1 \mid H \xleftarrow{R} \text{Binary}(n, k)] \right|.$$

The description of a Goppa code $\text{Goppa}(n, k)$ of length n and dimension k is to be found in [22].

B. McEliece scheme

[Key Generation] Let \mathcal{C} be a q -ary linear code t -correcting of length n and of dimension k . We denote $\mathcal{C}(n, k, d)$ a such code. Let G a generator matrix of \mathcal{C} . We will use an G' matrix such that :

$$G' = SHP \begin{cases} S \text{ is invertible} \\ P \text{ is a permutation matrix} \end{cases}$$

G' is public and its decomposition and a syndrome decoding algorithm for \mathcal{C} are secret. To be clearer, we recall the various sizes of the matrices :

S is $n - k \times n - k$, H is $n \times n - k$, P is $n \times n$.

[Encryption] Let $E_{q,n,t}$ be the space of \mathbb{F}_q^n words with Hamming weight t . For a chosen cleartext $x \in E_{q,n,t}$, y is the cryptogram corresponding to x if and only if $y = xG' + e$.

[Decryption] For $y = xG' + e$, the knowledge of the secret key allows :

1. to compute $u = yP^{-1}$;
2. to find u' from u thanks to a syndrome decoding algorithm;
3. to find $x = u'S^{-1}$.

The syndrome decoding algorithm can be, for instance, in the case of Goppa's codes, Patterson's algorithm (see part A).

C. Cryptanalytic Attacks

The security of code-based cryptosystems depends on the difficulty of the following two attacks :

- (i) *Structural Attack* : Recover the secret transformation and the description of the secret code(s) from the public matrix.
- (ii) *Ciphertext-Only Attack* : Recover the original message from the ciphertext and the public key.

C.1 Structural Attack

While no efficient algorithm for decomposing G' into (S, G, P) has been discovered yet [24], a structural attack has been discovered in [21]. This attack reveals part of the structure of a so-called weak G' where 'weak' means that G' has been generated from a binary Goppa polynomial in a special manner. However, this attack can be avoided simply by not using such weak public keys.

Structural attacks aim at recovering the structure of the permuted code, i.e. recovering the permutation from the code and its permuted version. The underlying problem is the equivalence of codes. This problem was considered by Sendrier for which he gave a nice solution : the Support Splitting Algorithm [26].

The complexity of this algorithm is in $\mathcal{O}(2^{\text{dimension}(\mathcal{C} \cap \mathcal{C}^\perp)})$ where \mathcal{C}^\perp is the dual of the code \mathcal{C} . This means that in order to resist the attack one gets two options : either starting from a large family of codes with arbitrary small hulls (the intersection of \mathcal{C} and \mathcal{C}^\perp) or starting from a small family of codes but with a large hull.

For instance the choice of Goppa codes corresponds to the first possibility.

C.2 Ciphertext-Only Attack

A first analysis using the Information-Set-Decoding was done by McEliece, then by Lee and Brickell, Stern and Leon and lastly by Canteaut and Chabaud (see [6] for all references).

The Information-Set-Decoding Attack is one of the known general attacks (i.e., not restricted to specific codes) and seems to have the lowest complexity.

One tries to recover the k information symbols as follows : the first step is to pick k of the n coordinates randomly in the hope that none of the k are in error. We then try to recover the message by solving the $k \times k$ linear system (binary or over \mathbb{F}_q). Let G'_k, c_k and z_k denote the k columns picked from G' , c and z , respectively. They have the following relationship

$$c_k = mG'_k + z_k.$$

If $z_k = 0$ and G'_k is non-singular, m can be recovered by

$$m = c_k G'^{-1}_k.$$

The computation cost of this version is $T(k) \times P_{n,k,t}$, where

$$P_{n,k,t} = \prod_{i=0}^{k-1} \left(1 - \frac{t}{n-i}\right).$$

The quantity $T(k)$ in the average work factor is the number of operations required to solve a $k \times k$ linear system over \mathbb{F}_q . As mentioned in [23], solving a $k \times k$ binary system takes about k^3 operations. Over \mathbb{F}_q , it would require at least $(k \times \log_2 q)^3$ operations.

All the papers which improve the complexity only impact the cost of the Gaussian elimination. In the best improvement by Canteaut and Chabaud [6] a good approximation of the cost besides the probability factor can be taken roughly in $(k \times \log_2 q)^2$.

Apart from these general attacks there are some attacks targeting McEliece cryptosystem using specific codes (see [28], [21], [4], [13] for example).

IV. SIGNATURE SCHEME OF COURTOIS, FINIASZ AND SENDRIER (OR CFS SCHEME)

BEFORE describing the CFS scheme we first recall the Niederreiter public key cryptosystem.

A. Niederreiter encryption scheme

[Key Generation] Let \mathcal{C} be a binary linear code t -correcting of length n and of dimension k . Let H a parity check matrix of \mathcal{C} . We will use an \tilde{H} matrix such that :

$$\tilde{H} = QHP \begin{cases} Q \text{ is invertible} \\ P \text{ is a permutation matrix} \end{cases}$$

\tilde{H} is public and its decomposition and a syndrome decoding algorithm for \mathcal{C} are secret.

To be clearer, we recall the various sizes of the matrices : Q is $n-k \times n-k$, H is $n \times n-k$, P is $n \times n$.

Let $E_{q,n,t}$ be the space of \mathbb{F}_q^n words with Hamming weight t .

[Encryption] For a chosen cleartext x in $E_{q,n,t}$, y is the cryptogram corresponding to x if and only if $y = \tilde{H}x^T$.

[Decryption] For $y = \tilde{H}x^T$, the knowledge of the secret key allows :

1. to compute $Q^{-1}y$ ($= HPx^T$);

2. to find Px^T from $Q^{-1}y$ thanks to a syndrome decoding algorithm;
3. to find x applying P^{-1} to Px^T .

The syndrome decoding algorithm can be, for instance, in the case of Goppa's codes, Patterson's algorithm (see part A).

The McEliece or the Niederreiter schemes are not naturally invertible, i.e. if one starts from a random element y of \mathbb{F}_2^n and a code $\mathcal{C}[n, k, d]$ that we are able to decode up to $d/2$, it is almost sure that we won't be able to decode y into a codeword of \mathcal{C} . This comes from the fact that the density of the whole space that is decodable is very small.

B. CFS signature scheme

The idea of the CFS scheme is to find parameters $[n, k, d]$ that make successful the strategy of picking up random elements until one is able to decode it with high probability. More precisely, given M a message to sign and h a hash-function with range $\{0, 1\}^{n-k}$, we try to find a way to build $s \in \mathbb{F}_2^n$ of given weight t such that $h(M) = \tilde{H}s^T$. For $\text{Decode}(\cdot)$ a decoding algorithm, the CFS scheme works as follows :

[Key Generation]

1. Select n, k and t according to the security parameter κ .
2. Pick a random parity check matrix \tilde{H} of a (n, k) -binary Goppa code decoding t errors.
3. Choose a random $(n-k) \times (n-k)$ non-singular matrix Q , a random $n \times n$ permutation matrix P and a hash-function $h : \{0, 1\}^* \longrightarrow \mathbb{F}_2^{n-k}$.
4. The public key is $H = Q\tilde{H}P$ and the private key is (Q, \tilde{H}, P) .
5. Set $t = \frac{n-k}{\log_2 n}, i = 0$.

[Sign]

1. $i \leftarrow i + 1$
2. $x' = \text{Decode}_{\tilde{H}}(Q^{-1}h(m\|i))$
3. if no x' was found go to 1
4. output $(i, x'P)$

[Verify] Compute $s' = Hx'^T$ and $s = h(m\|i)$. The signature is valid if s and s' are equal.

We get at the end an $\{s, j\}$ couple, such that :

$$h(M \oplus j) = \tilde{H}s^T.$$

Let us notice that we can suppose that s has weight $t = [d/2]$. In [12], a proof of security in the Random Oracle Model for a modified version of the CFS scheme is given. We use the modified CFS scheme described there, and named as **mCFS**, as a building block for our scheme. The **mCFS** scheme is explained next.

C. Modified CFS signature scheme

[Key Generation]

1. Select n, k and t according to κ .
2. Pick a random parity check matrix \tilde{H} of a (n, k) -binary Goppa code decoding t errors.

3. Choose a random $(n-k) \times (n-k)$ non-singular matrix Q , a random $n \times n$ permutation matrix P and a hash-function $h : \{0, 1\}^* \longrightarrow \mathbb{F}_2^{n-k}$.
4. The public key is $H = Q\tilde{H}P$ and the private key is (Q, \tilde{H}, P) .
5. Set $t = \frac{n-k}{\log_2 n}$.

[Sign]

1. $i \xleftarrow{R} \{1, \dots, 2^{n-k}\}$
2. $x' = \text{Decode}_{\tilde{H}}(Q^{-1}h(m\|i))$
3. if no x' was found go to 1
4. output $(i, x'P)$

[Verify] Compute $s' = Hx'^T$ and $s = h(m\|i)$. The signature is valid if s and s' are equals.

V. STERN'S PROTOCOL

STERN'S scheme is an interactive zero-knowledge protocol which aims at enabling a *prover* P to identify himself to a *verifier* V .

Let n and k be two integers such that $n \geq k$. Stern's scheme assumes the existence of a public $(n-k) \times n$ matrix \tilde{H} defined over the two elements field \mathbb{F}_2 . It also assumes that an integer $t \leq n$ has been chosen. For security reasons (discussed in [30]) it is recommended that t is chosen slightly below the so-called Gilbert-Varshamov bound (see [22]). The matrix \tilde{H} and the weight t are protocol parameters and may be used by several (even numerous) different provers

Each prover P receives a n -bit secret key SK (also denoted by s if there is no ambiguity about the prover) of Hamming weight t and computes a *public identifier* PK such that $i_P = \tilde{H}SK^T$. This identifier is calculated once in the lifetime of \tilde{H} and can thus be used for several identifications. When a user P needs to prove to V that he is indeed the person associated to the public identifier PK , then the two protagonists perform the following protocol where h denotes a standard hash-function :

[Commitment Step] P randomly chooses $y \in \mathbb{F}_2^n$ and a permutation σ of $\{1, 2, \dots, n\}$. Then P sends to V the commitments c_1, c_2 and c_3 such that :

$$c_1 = h(\sigma\|\tilde{H}y^T); \quad c_2 = h(\sigma(y)); \quad c_3 = h(\sigma(y \oplus SK)),$$

where $h(a\|b)$ denotes the hash of the concatenation of the sequences a and b .

[Challenge Step] V sends $b \in \{0, 1, 2\}$ to P .

[Answer Step] Three possibilities :

- if $b = 0$: P reveals y and σ .
- if $b = 1$: P reveals $(y \oplus SK)$ and σ .
- if $b = 2$: P reveals $\sigma(y)$ and $\sigma(SK)$.

[Verification Step] Three possibilities :

- if $b = 0$: V verifies that c_1, c_2 are correct.
- if $b = 1$: V verifies that c_1, c_3 are correct.

- if $b = 2$: V verifies that c_2, c_3 are correct, and that the weight of $\sigma(s)$ is t .

[Soundness Amplification Step] Iterate the above steps until the expected security level is reached.

During the fourth Step, when b equals 1, it can be noticed that $\tilde{H}y^T$ derives directly from $\tilde{H}(y \oplus SK)^T$ since we have :

$$\tilde{H}y^T = \tilde{H}(y \oplus SK)^T \oplus PK = \tilde{H}(y \oplus SK)^T \oplus \tilde{H}SK^T .$$

As proved in [30], the protocol is zero-knowledge and for a round iteration, the probability that a dishonest person succeeds in cheating is $(2/3)$. Therefore, to get a confidence level of β , the protocol must be iterated a number of times k such that $(2/3)^k \leq \beta$ holds. When the number of iterations satisfies the last condition, then the security of the scheme relies on the NP complete problem SD.

By virtue of the so-called Fiat-Shamir Paradigm [15], it is possible to convert Stern's Protocol into a signature scheme, but the resulting signature size is long (about 140-kbit long for 2^{80} security). Notice that this is large in comparison with classical signature schemes, but it is more or less close to the size of many files currently used in everyday life.

VI. NEW IDENTITY-BASED IDENTIFICATION SCHEME FROM STERN-NIEDERREITER PROTOCOLS

WE describe now the first code-based identity-based identification method. The prover is identifying herself to the verifier. Let id_S, id_P be the prover and of the identifier identities respectively.

[Master key generation] Let $\mathcal{C}, H, \tilde{H} = QHP$ the output of the key generation algorithm of the CFS signature scheme in Section IV. Let h a hash function mapping to $\{0,1\}^{n-k}$. \tilde{H} is made public, but the decomposition of \tilde{H} is a secret of the authority.

[Key extraction] On inputs the the decomposition of \tilde{H} and the user's identity id_P the goal of the key extraction algorithm is to output $s \in E_{q,n,t}$ such that $h(id_P) = \tilde{H}s^T$. However $h(id_P)$ might not be in the target of $x \rightarrow \tilde{H}x^T$. That is to say that $h(id_P)$ is not necessarily in the space of decodable elements of \mathbb{F}_q^n . That problem can be solved thanks to the following algorithm. Given $Decode(\cdot)$ a decoding algorithm for the hidden code :

1. $i \xleftarrow{R} \{1, \dots, 2^{n-k}\}$
2. $x' = Decode_{\tilde{H}}(Q^{-1}h(id_P \parallel i))$
3. If no x' was found go to 1
4. output $(i, x'P)$

We get at the end a couple $\{s, j\}$, such that $h(id_P \parallel j) = \tilde{H}s^T$. We can note that we have s of weight t or less.

[Interactive identification] We use a slight derivation of Stern's protocol. We suppose that the prover obtained a couple $\{s, j\}$ verifying $h(id_P \parallel j) = \tilde{H}s^T$. $h(id_P \parallel j)$ is set to be the prover's public key. Identification is then performed by modifying Stern's protocol with respect to the public key $h(id_P \parallel j)$. Details follow.

[Commitment Step] P chooses randomly any word y of n bits and a permutation σ of $\{1, 2, \dots, n\}$. Then P sends to $S : c_1, c_2, c_3, j$ such that :

$$c_1 = h(\sigma \parallel \tilde{H}y^T); c_2 = h(\sigma(y)); c_3 = h(\sigma(y \oplus s))$$

[Challenge Step] S sends $b \in \{0, 1, 2\}$ to P .

[Answer Step] Three possibilities :

- if $b = 0$: P reveals y and σ .
- if $b = 1$: P reveals $(y \oplus s)$ and σ .
- if $b = 2$: P reveals $\sigma(y)$ and $\sigma(s)$

[Verification Step] Three possibilities :

- if $b = 0$: S verifies that the c_1, c_2 received at the second round are correct.
- if $b = 1$: S verifies that the c_1, c_3 received at the second round are correct. For c_1 we can note that $\tilde{H}y^T$ derives directly from $\tilde{H}(y \oplus s)^T$ by :

$$\tilde{H}y^T = \tilde{H}(y \oplus s)^T \oplus \tilde{H}s^T$$

- if $b = 2$: S verifies that the c_2, c_3 received at the second round have really been honestly calculated, and that the weight of $s \parallel \sigma$ is t .

[Soundness Amplification Step] Iterate the commitment, challenge, answer and verification steps until the expected security is reached.

Thanks to the Fiat-Shamir heuristic [15] it is possible to derive an identity-based signature scheme from the above identity-based identification scheme. Since this is a well-known cryptographic result, we refer the reader to [15], [1] for details.

VII. PROVING SECURITY OF mCFS-Stern IBI SCHEME

Theorem 1 *The IBI scheme from Section VI is secure in the sense of imp-pa if the BD and CD problems are hard to solve.*

Proof: A security reduction is obtained by adapting the proofs by Dallot [12] and Stern [31] to our setting. We build the proof following a sequence of games **Game 0**, **Game 1**, ... **Game 0** is the original attack game, i.e the standard imp-pa game. Successive games are obtained by small modifications of the preceding games, in such a way that the difference of the adversarial advantage in consecutive games is easily quantifiable. To compute this difference, the following lemma is used :

Lemma 1 *Let X_i, X_{i+1}, B be events defined in some probability distribution, and suppose that $X_i \wedge \neg B \Leftrightarrow X_{i+1} \wedge \neg B$. Then $|\Pr[X_i] - \Pr[X_{i+1}]| \leq \Pr[B]$.*

Let q_h, q_E, q_C denote the maximum number of queries that adversary \mathcal{A} makes to the hash, user keys and conversation oracles.

We want to show there exists adversaries \mathcal{C}, \mathcal{D} that break the BD and CD problems respectively.

To answer hash, user key and conversation queries, three lists $h^{\text{list}}, UK^{\text{list}}$ and Λ are maintained. If there is no value associated with an entry in a list, we denote its output by \perp . The list h^{list} consists of tuples of the form (x, s) indexed by (id, i) , where i is an index in $\{1, \dots, 2^{n-k}\}$, id is an identity, and $\tilde{H}s^T = x = h(id, i)$ if $x \neq \perp \neq s$. The list UK^{list} , consists of entries of the form $(id, sk[id])$. The list Λ contains indexes $\Lambda(m)$ associated to a message m , for which the simulator is able to produce a signature on $h(m, \Lambda(m))$.

Game 0. This the standard imp-pa game. The master public and secret keys are obtained by running algorithm $\text{Gen}_{\text{mCFS}}(1^\kappa)$. In particular, the master public key $\tilde{H} = QHP$ plus a hash-function $h : \{0, 1\}^* \rightarrow \mathbb{F}_2^{n-k}$, and the master secret key is (Q, H, P) , where $H \xleftarrow{R} \text{Goppa}(n, k)$, Q is a non-singular $(n-k) \times (n-k)$ matrix and P is a $n \times n$ permutation matrix. Therefore $\Pr[X_0] = \text{Adv}_{IBI, \mathcal{A}}^{\text{imp-pa}}$.

Game 1. (Simulation of hash and user key queries) We change the way in which hash and user key extraction queries are answered. For hash queries of the form (id, i) , there are two situations, depending on whether $i = \Lambda(id)$. If this is the case, a decodable syndrome $x = \tilde{H}s^T$ is given as the output, and the corresponding code-word s is stored, i.e. h^{list} is updated with (x, s) in the entry indexed by (id, i) . If $i \neq \Lambda(id)$ hash queries are simulated by taking a random element in \mathbb{F}_2^{n-k} , and then these queries are distributed as with a random oracle. Details are shown in Figure 1.

On the other hand, user key queries on id are answered by choosing the special index $\Lambda(id)$ at random, calling the hash oracle on $(id, \Lambda(id))$ and outputting (s, i) as the resulting user secret key. Details are shown in Figure 2.

At the end of the simulation, the random oracle h has output $q_h + q_E + 1$ syndromes. Some of them are produced with the special index $i = \Lambda(id)$; these syndromes are not distributed uniformly at random in \mathbb{F}_2^{n-k} , instead they have been modified as to enable responding user secret key queries. It might be then the case that adversary \mathcal{A} queried h on some pair (id, j) such that later j is set to $\Lambda(id)$. This will cause an incoherence, since then the output $h(id, j)$ will be a random syndrome, instead of a decodable syndrome. The latter happens with probability at most $\frac{q_E}{2^{n-k}}$ (the indexes $\Lambda(id)$ are only defined when answering key extraction queries). Therefore,

$$|\Pr[X_0] - \Pr[X_1]| \leq \frac{q_E}{2^{n-k}}$$

Game 2. (Changing the master key generation algorithm) The key generation algorithm is changed so that $H \leftarrow$

$\text{Binary}(n, k)$. Then,

$$|\Pr[X_2] - \Pr[X_1]| \leq \text{Adv}_{\mathcal{D}}^{\text{CD}}(n)$$

where \mathcal{D} is an algorithm that simulates the environment of Game 2 for \mathcal{A} if $H \leftarrow \text{Goppa}(n, k)$ and outputs $d = 1$ if \mathcal{A} successfully impersonates the target identity id^* , and $d = 0$ otherwise; and \mathcal{D} simulates the environment of Game 3 for \mathcal{A} if $H \leftarrow \text{Binary}(n, k)$ and outputs $d = 1$ if \mathcal{A} successfully impersonates the target identity id^* , and $d = 0$ otherwise. It is easy to see that

$$\Pr[H \xleftarrow{R} \text{Goppa}(n, k) : \mathcal{D}(H) = 1] = \Pr[X_2].$$

and

$$\Pr[H \xleftarrow{R} \text{Binary}(n, k) : \mathcal{D}(H) = 1] = \Pr[X_3].$$

```

Input: A pair  $(id, i)$ 
Output: A syndrome  $s$ 
 $(s, x) \leftarrow h^{\text{list}}(m, i);$ 
if  $i \neq \Lambda(id)$  then
  if  $s = \perp$  then
     $x \xleftarrow{R} \mathbb{F}_2^{n-k};$ 
     $h^{\text{list}}(id, i) \leftarrow (x, \perp);$ 
  end
  return  $h(id, i) = x;$ 
else
  if  $x = \perp$  then
     $s \xleftarrow{R} \{w \in \mathbb{F}_2^n \mid \text{wt}(w) \leq t\};$ 
     $x \leftarrow \tilde{H}s^T;$ 
     $h^{\text{list}}(id, i) \leftarrow (x, s);$ 
  end
  return  $h(id, i) = x;$ 
end

```

Fig. 1 - Simulation of hash queries

```

Input: An identity  $id$ 
Output: A user secret key  $(s, id)$ 
if  $\Lambda(id) = \perp$  then
   $\Lambda(id) \xleftarrow{R} \{1, \dots, 2^{n-k}\};$ 
end
 $(x, s) \leftarrow h(id, \Lambda(id));$ 
 $i \leftarrow \Lambda(id);$ 
 $\Lambda(id) \leftarrow \perp;$ 
return  $sk[id] = (s, i);$ 

```

Fig. 2 - Simulation of user key queries

Game 3. (Guessing the target identity) A random index $j^+ \xleftarrow{R} \{1, \dots, q_h + q_E + q_C\}$ is taken. The j^+ -th hash query (id^+, i^+) to is set to be $Q(x^+)^T$, where $x^+ \xleftarrow{R} \mathbb{F}_2^{n-k}$, i.e. $h(id^+, i^+) = Q(x^+)^T$. The probability space is not modified, since $x^+ \xleftarrow{R} \mathbb{F}_2^{n-k}$ and Q is non-singular, and therefore $\Pr[X_3] = \Pr[X_2]$.

Game 4.(Abort the game)

Let (id^*, i^*) be the target identity and target index that \mathcal{A} impersonates. If $id^* \neq id^+$ or $i^* \neq i^+$ then the challenger aborts the game. Since Game 4 is obtained by conditioning Game 3 on an independent event of probability $\frac{1}{q_H + q_E + 1}$ we obtain

$$\Pr[X_4] = \frac{\Pr[X_3]}{q_H + q_E + 1}$$

Game 5. (Answering conversation queries on the target identity id^*) We have to answer conversation queries on id^* without knowing the code word s^* corresponding to $h(id^*, i^*) = x^*$, i.e. s^* such that $x^* = \tilde{H}s^+$ and $x^* = Q(x^+)^T$. We can answer these queries in expected polynomial time by using the algorithm in Theorem 3 in [31]. Roughly, the algorithm uses a resettable simulation [18]. At the beginning of each iteration of the basic identification protocol, the algorithm chooses at random one out of three cheating strategies, where each strategy allows to successfully interact with a cheating verifier \overline{CV} with probability 2/3. In case the algorithm can not successfully interact with \overline{CV} , it resets the adversary \mathcal{A} for the current round (see [31] for details). All in all, the probability space is not modified, and then $\Pr[X_5] = \Pr[X_4]$.

Theorem 1 in [31] implies that an adversary \mathcal{A} impersonating the user with identity id^* when running k rounds of the basic protocol and with advantage $(2/3)^k + \epsilon_1$ for a non-negligible $\epsilon_1 > 0$, can be converted into a PPT algorithm computing s^* such that $\tilde{H}(s^*)^T = x^*$ with probability $\epsilon_1^3/10$. A basic calculation shows that $(s^+)^T = P(s^*)^T$ is a solution to the BD problem with inputs $H \xleftarrow{R} \text{Binary}(n, k)$ and $x^+ \xleftarrow{R} F_2^{n-k}$. Let \mathcal{C} be an algorithm that simulates Game 5 for the impersonating adversary \mathcal{A} using the input of the BD problem. Then,

$$\text{Adv}_{\mathcal{C}}^{\text{BD}} \geq \frac{(\Pr[X_5] - (2/3)^k)^3}{10}$$

Collecting all the probabilities

$$\begin{aligned} (2/3)^k + \epsilon &\leq \text{Adv}_{\mathcal{C}^{\text{BD}}, \mathcal{A}}^{\text{imp-pa}} \\ &\leq \frac{q_E}{2^{n-k}} + \text{Adv}_{\mathcal{D}}^{\text{CD}}(n) + \Pr[X_5](q_h + q_E + 1) \\ &\leq \frac{q_E}{2^{n-k}} + \text{Adv}_{\mathcal{D}}^{\text{CD}}(n) + \\ &\quad \left((\text{Adv}_{\mathcal{C}}^{\text{BD}})^{1/3} + (2/3)^k \right) 10^{1/3} (q_h + q_E + 1) \end{aligned}$$

and then

$$\begin{aligned} \epsilon &\leq \frac{q_E}{2^{n-k}} + \text{Adv}_{\mathcal{D}}^{\text{CD}}(n) + \\ &\quad \left((\text{Adv}_{\mathcal{C}}^{\text{BD}})^{1/3} + \left(1 - \frac{1}{\sqrt[3]{10}}\right) \left(\frac{2}{3}\right)^k \right) 10^{1/3} (q_h + q_E + 1) \end{aligned}$$

The latter equation can be read as follows : a successful impersonating adversary with advantage $(\frac{2}{3})^k + \epsilon$ implies a successful adversary against the BD or CD problems. ■

VIII. EFFICIENCY ANALYSIS

We deal here with the security our protocol and its practicality. Let us remind that in the case of Niederreiter's cryptosystem, its security relies on the hardness of decoding of a linear code (see section III).

A. Parameters and security of the scheme

The protocol has two parts : in the first part one inverts the syndrome decoding problem for a matrix \tilde{H} in order to construct a private key for the prover and in second part one applies Stern identification protocol with the same matrix \tilde{H} . This shows that the overall parameters of the scheme are equivalent to the security of the CFS scheme, since the security of the Stern scheme with the same matrix parameters is implicitly included in the signature scheme.

In particular the scheme has to fulfill two imperative conditions :

1. make the computation of $\{s, j\}$ (defined in advance) difficult without the knowledge of the description of H ,
2. make the number of trials to determine the correct j not too important in order to reduce the cost of the computation of s .

Following [11] the Goppa $[2^m, 2^m - tm, t]$ codes are a large class of codes which are compatible with condition 2. Indeed, for such a code, the proportion of the decodable syndromes is about $1/t!$ (which is a relatively good proportion). We also have to choose a relatively small t .

The $\{s, j\}$ production process will thus be iterated, about $t!$ times before finding the correct j . But each iteration forces to compute $D(h(id_P \| j))$.

The decoding of the Goppa codes consists of :

- computing a syndrome : $t^2 m^2 / 2$ binary operations;
- computing a locator polynomial : $6t^2 m$ binary operations;
- computing its roots : $2t^2 m^2$ binary operations.

We thus get a total cost for the computation of the prover's private key of about :

$$t! t^2 m^2 (1/2 + 2 + 6/m) \text{ binary operations}$$

The cost of an attack by decoding thanks to the *split syndrome decoding* is estimated to : $2^{tm(1/2+o(1))}$.

The choice of parameters will have to be pertinent enough to conciliate cost and security. Although less important, some sizes have also to remain reasonable : the length of $\{s, j\}$, the cost of the verification and the size of \tilde{H} that is for a Goppa code : $2^m t m$.

Following [11] we can for example take $t = 9$ and $m = 16$. The cost of the signature stays then relatively reasonable for a security of about 2^{80} . The others sizes remain in that context very acceptable.

B. Practical values

The big difference when using the parameters associated to the CFS scheme is that the code used is very long, 2^{16} against 2^9 for the basic Stern scheme, it dramatically develops communication costs.

In the next table we sum up for the parameters $m = 16$, $t = 9$ the general parameters of the IBI and IBS schemes.

public key	private key	matrix size
tm	tm	$2^m tm$
144	144	1 Mo

communication cost	key generation
$\approx 2^m \times \#rounds$	
500 Ko (58 rounds)	1 s

Practical values for the IBI scheme : $m = 16, t = 9$

signature length	key generation
$\approx 2^m \times \#rounds$	
2.2 Mo (280 rounds)	1 s

Practical values for the IBS scheme : $m = 16, t = 9$

Reduction of the size of the public matrix : At the difference of a pure signature scheme in which one wants to be able to sign fast, in our scheme the signature is only computed once for sending it to the prover, hence the time for signing may be judged less determinant and a longer time of signature may be accepted at the cost of reducing (a little) the parameters of the public matrix.

IX. CONCLUSION

IN this paper we present and prove secure a new identity-based identification scheme based on error-correcting codes. Our scheme combines two well known schemes by Courtois-Finiasz-Sendrier and Stern. It inherits some of their practical weaknesses, such as large system parameters. Interestingly the new scheme is one of the very few existing alternatives to number theory for identity-based cryptography, and we hope that it boosts future research on this area.

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